P105 TIME-DOMAIN RAY ASYMPTOTIC NEAR SINGULARITIES - ILLUSTRATION FOR A CAUSTIC CUSP

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Abstract

We propose an extension of the time-domain ray method that remains valid near caustics. It does not require complex ray tracing in the caustic shadow but needs higher travel-time derivatives to be calculated along the ray (this can be done by solving differential equations similar to dynamic ray-tracing system).

Introduction

Geometrical ray tracing, combined with asymptotic methods for estimating the wave amplitudes along the rays, is widely used in seismic studies. With the development of computers numerical methods become a reasonable tool for seismic wavefield modeling replacing the ray method. But there are areas where the ray theory is necessary, e.g., it forms the basis and is an essential part of the Kirchhoff migration and the AVO analysis. In both cases it is important to calculate amplitude dynamics properly (including points near caustics). Thus it is an important issue: extending the ray theory to make it work near caustics. A lot had been done in this field (e.g. see [Hanyga and Helle, 1995]). Proposed techniques may be called global asymtotics as they use information from several rays (coalescing at caustic) to compute the wave field in singular point. In the caustic shadow one should trace complex rays. We propose a variant of local asymptotic that allows computing seismic signal along one separate ray and remains valid at caustics. Standard ray method procedure is as follows: calculation of the second travel-time derivatives along the ray provides ray amplitudes in regular points (dynamic ray tracing). Similarly calculation of the higher order travel-time derivatives along the ray provides seismic signal description that remains valid near caustics (third travel-time derivatives are required near a simple caustic, forth near the cusp and etc.). These derivatives may be calculated as a solution of linear differential equations.

Theory

The detailed method description may be found in [Goldin and Duchkov, 2000] and [Duchkov and Goldin, 2001]. In asymptotic ray theory *P*- and *S*-waves are computed independently. If $\mathbf{x}_1$ is a regular point of the ray we use the zero's term of ray series to compute seismic signal:

$$\mathbf{u}(\mathbf{x},t) = \mathbf{U}_0(\mathbf{x}) R_q^{(+)}\left(t - \tau(\mathbf{x})\right) \tag{1}$$

where $\mathbf{u}(\mathbf{x},t)$ is the displacement vector, $\tau(\mathbf{x})$ and $\mathbf{U}_0(\mathbf{x})$ denote travel-time and amplitude (they can be found using standard procedure of ray tracing and dynamic ray tracing). Discontinuous function $R_q^{(+)}(t)$ (an ideal wave or time-domain asymptotic) is defined as:

$$R_q^{(+)}(t) = \begin{cases} t_+^q / \Gamma(q+1), & q \neq -1,-2,... \\ \delta^{(-q+1)}(t), & q = -1,-2,... \end{cases} \quad (2)$$

where $\delta(t)$ is Dirac δ-function. Now if $\mathbf{x}_1$ is singular then we go back to the regular point $\mathbf{x}_0$ of the same ray and construct Kirchhoff-type integral representation of the wave field:

$$\mathbf{u}(\mathbf{x},t) = \iint_S \left\{\mathbf{G}(\mathbf{x}_1) * \mathrm{T}_\mathbf{n}[\mathbf{u}^{(in)}] - \mathbf{u}^{(in)} * \mathrm{T}_\mathbf{n}[\mathbf{G}(\mathbf{x}_1)]\right\} dS, \quad (3)$$

where $\mathbf{G}(\mathbf{x}_1)$ is the Green's tensor for the point source placed at point $\mathbf{x}_1$, $\mathbf{u}^{(in)}$ is the wavefield calculated at point $\mathbf{x}_0$, $\mathrm{T}_\mathbf{n}[\,]$ is the Cauchy stress operator for surface element dS with normal **n,** surface S is transversal to ray and contains $\mathbf{x}_0$. For asymptotic solutions integrals (3) may be reduced to time-domain analogues of oscillatory integrals:

$$\iint \mathbf{L}(x,y)\delta\left(t - \tau(x,y)\right) dxdy\,, \quad (4)$$

where integration is local in the vicinity of zero. Thus we do not need function τ but some of its derivatives at zero (third derivatives for simple caustic and etc.). Derivatives of $\mathbf{L}$ and τ are found from $\mathbf{G}(\mathbf{x}_1)$ and $\mathbf{u}^{(in)}$ at point $\mathbf{x}_0$.

Examples

We used synthetic examples to illustrate the technique application. In the Fig. 1 one can see the caustic cusp (bold line) that separates "illuminated zone" (three rays hit every point) from the caustic "shadow" (one ray hits every point). A seismic ray propagating in homogeneous medium (bold arrow) from shadow into illuminated zone. Seismic signal was calculated at several points of the ray (main component of the *P*-wave). Upper panels correspond to the composite signal computed from (4). Here singularity is an idealized model of the wave. This representation may be considered as a time-domain variant of a Piercey function (known for harmonic fields). It is mainly illustrative showing how three different signals (singularities) interfere in the vicinity of a caustic cusp. After convolution with the source function we get a synthetic seismic signal (lower panels) and see how caustic distorts its form. It is interesting to look at first point of the ray situated in a shadow of the caustic cusp. Smooth maximum in the composite signal (upper left panel) is a result of energy dissipation from illuminated area into shadow. Example of synthetic seismograms is shown in Fig. 2. We consider *P*-wave reflected from a curvilinear boundary. A set of reflected rays has an envelope, i.e., caustic cusp (see upper panel). We put a line of receivers (bold segment) in the shadow of the caustic cusp. Only one ray hits every receiver but caustic causes standard ray method to break down (panel (a)). Application of our integral formulas provides correct signal form in this case (panel (b)). Note that our method accounts for energy dissipation into the caustic shadow without tracing complex rays or Gaussian beams.

Conclusions

Proposed extension of the ray theory provides wave description that remains valid near caustics. It works well in time domain avoiding forward and inverse Fourier transform. Synthetic tests show that it works well in the shadow zone of the caustic cusp with out complex ray tracing or Gaussian beams summation. It requires higher travel-time derivatives to be calculated along the ray (this may be done by solving special differential equations).

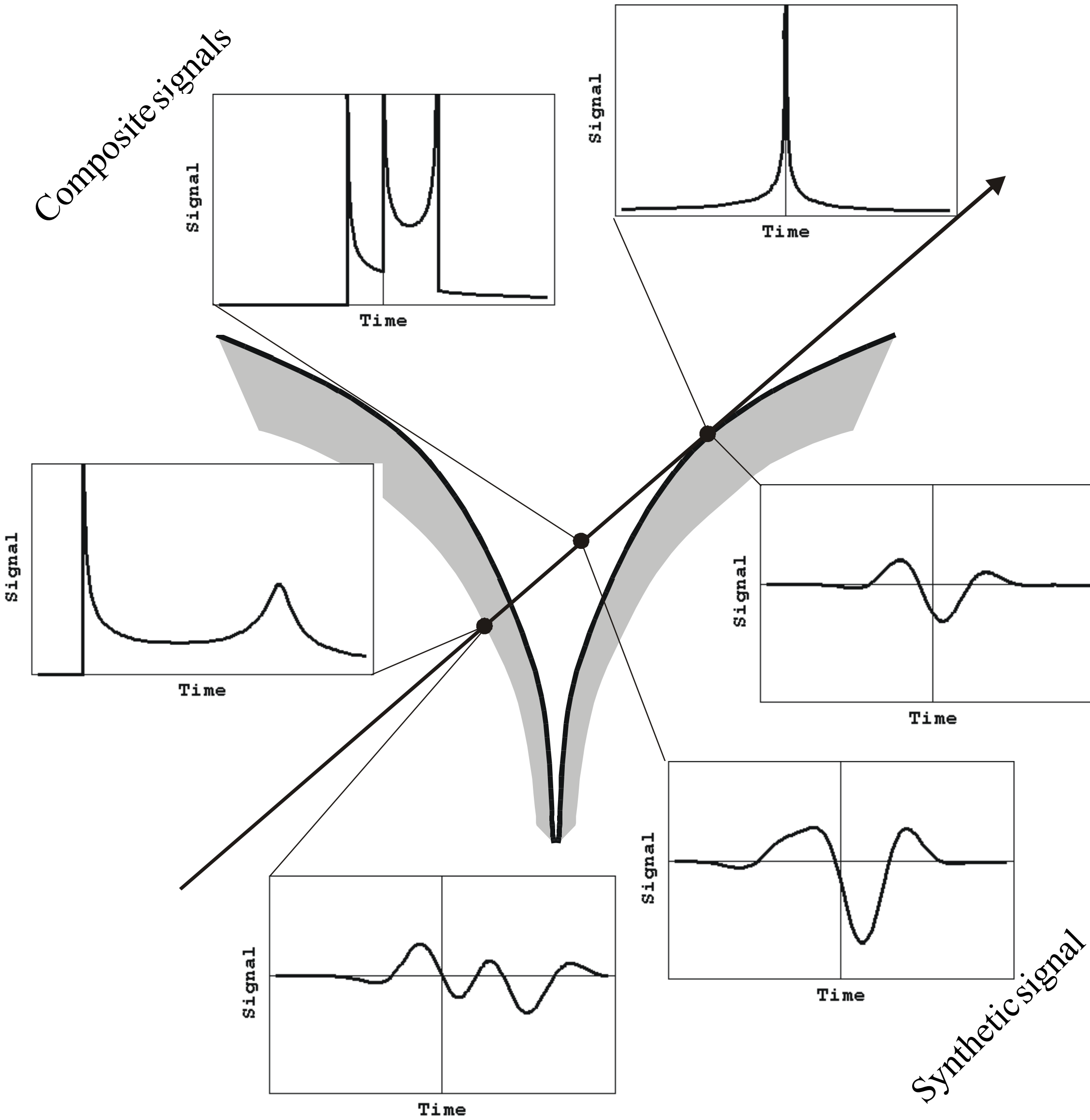

Fig. 1. Seismic signal calculated at different points of the ray near caustic cusp (see description in the text).

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References

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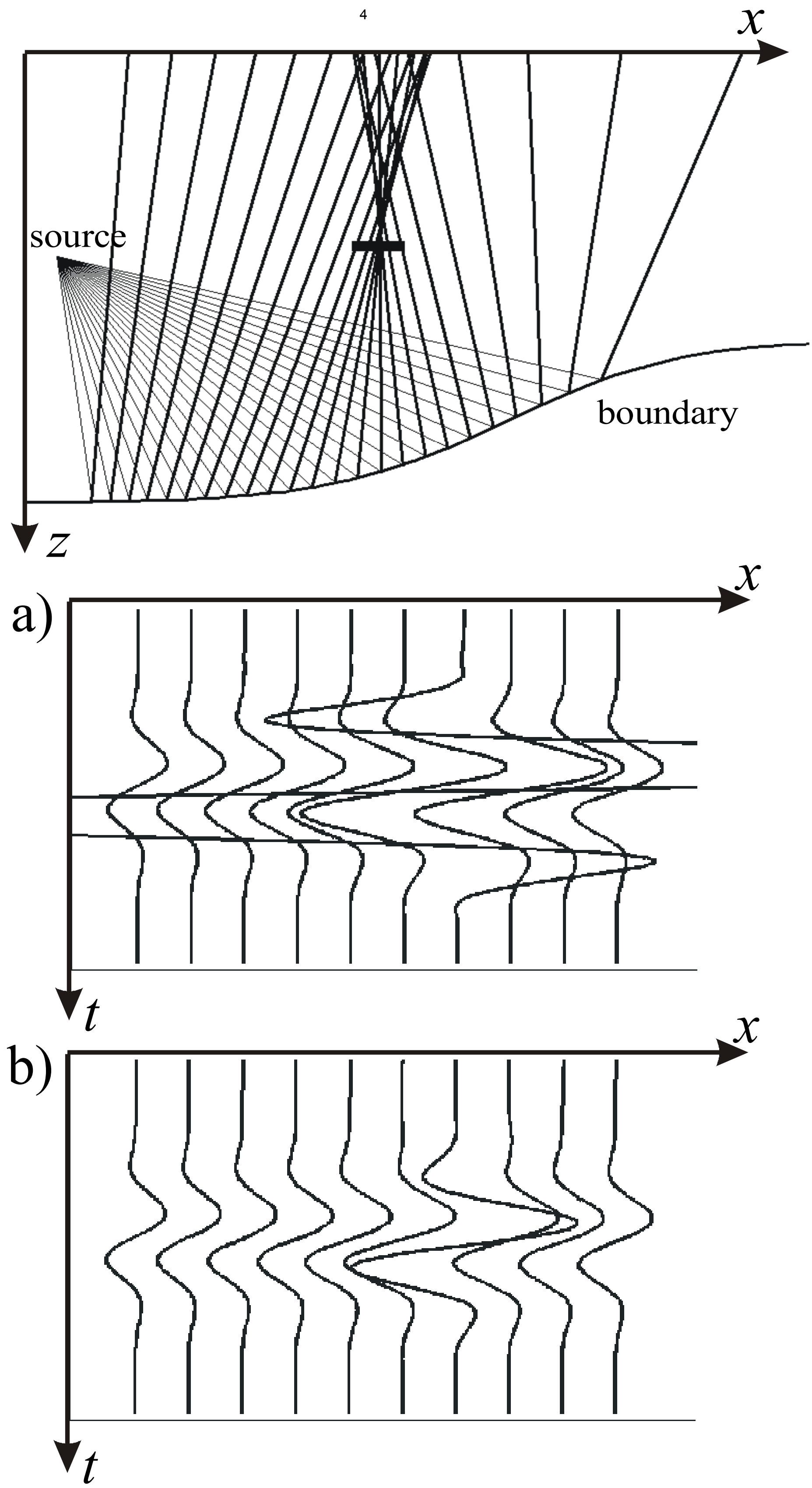

Fig. 2 Synthetic seismograms for reflected *P*-wave, vertical component (receiver line – bold segment in the upper panel). Standard ray method (a) and our approach (b).